Let B = {x<sub>1</sub>,..., x<sub>n</sub>} be a finite, ordered basis of a vector space V. Any vector v ∈ V can be written uniquely as

$$\alpha_1 \mathbf{x}_1 + \cdots + \alpha_n \mathbf{x}_n.$$

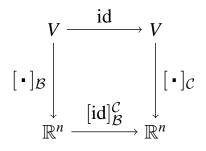
The vector  $[\mathbf{v}]_{\mathcal{B}} = \langle \alpha_1, \dots, \alpha_n \rangle \in \mathbb{R}^n$  is called the **coordinate** representation of **v** with respect to the ordered basis  $\mathcal{B}$ .

If V is an n-dimensional vector space and B is any ordered basis of V, then coordinate representation gives an isomorphism from V to ℝ<sup>n</sup>.

## **Transition Matrices**

Let *V* be a finite dimensional vector space. Let  $\mathcal{B} = {\mathbf{x}_1, ..., \mathbf{x}_n}$  and  $\mathcal{C} = {\mathbf{y}_1, ..., \mathbf{y}_n}$  be bases of *V*. Let id :  $V \to V$  be the identity function.

- The **transition matrix** matrix  $[id]_{\mathcal{B}}^{\mathcal{C}}$  is the  $n \times n$  matrix whose  $j^{\text{th}}$  column is the vector  $[\mathbf{x}_i]_{\mathcal{C}}$ .
- Theorem 4.26.1: For all  $\mathbf{x} \in V$ , we have  $[\mathrm{id}]_{\mathcal{B}}^{\mathcal{C}}[\mathbf{x}]_{\mathcal{B}} = [\mathbf{x}]_{\mathcal{C}}$ .



• **Theorem 4.26.2:** The matrix  $[id]_{\mathcal{B}}^{\mathcal{C}}$  is invertible, and  $([id]_{\mathcal{B}}^{\mathcal{C}})^{-1} = [id]_{\mathcal{C}}^{\mathcal{B}}$ .

## Matrix Representations of Linear Transformations

Let *V* and *W* be finite dimensional vector spaces.

Let  $\mathcal{B} = {\mathbf{x}_1, \ldots, \mathbf{x}_n}$  be an ordered basis of *V*, and let

 $C = {\mathbf{y}_1, \ldots, \mathbf{y}_m}$  be an ordered basis of *W*.

Let  $f: V \to W$  be a linear transformation.

- We define  $[f]_{\mathcal{B}}^{\mathcal{C}}$  to be the matrix whose columns are  $[f(\mathbf{x}_1)]_{\mathcal{C}}$ ,  $[f(\mathbf{x}_2)]_{\mathcal{C}}, \ldots [f(\mathbf{x}_n)]_{\mathcal{C}}$ .
- **Theorem 4.33:** With the above notation, for all  $\mathbf{x} \in V$ , we have

## **Transition Matrices**

**Theorem 4.35:** Let *V* be a finite dimensional vector space, with ordered bases  $\mathcal{B}$  and  $\mathcal{C}$ . Let  $f : V \to V$  be a linear transformation, and let  $P = [\operatorname{id}]_{\mathcal{C}}^{\mathcal{B}}$ . Then